HIV/AIDS and Fatalism: Should Prevention Campaigns Disclose the Transmission Rate of HIV?

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Abstract

Among non-specialists, the estimates of the HIV transmission rate are generally upwardly biased. This overestimation may be perceived as a godsend, as it increases the incentives to have protected sexual relationships. However, a pernicious effect may counterbalance this positive effect. Combined with the overestimation of the transmission rate, an occasional unprotected sexual encounter may induce the feeling that “the die is cast”, and hence lead to a permanent neglect of condom use. This paper proposes a theoretical model predicting this fatalistic reaction in both regular and casual relationships. Simulations of the model show that the expected transmission rate which would maximize condom use in high-risk populations ranges between 4.8% and 24.7%. Disclosing a transmission rate below 4.8% would increase risky practices, especially in subpopulation where casual sex is common and where partners are at low risk of infection.

Keywords: HIV/AIDS, transmission rate, prevention, risk perception, condom, Burundi

JEL Classification: I15, I18, D81, D83, D84

1. Introduction

In 2009, an estimated 33.3 million people were living with HIV (UNAIDS, 2010). Despite increasing resources dedicated to contain the spread of the epidemic, UNAIDS estimated that 2.6 million people became newly infected with HIV in 2009. Condom is one of the most popular tools to prevent new infections. However, “no clear examples have emerged yet of a country that has turned back a generalize epidemic primarily by means of condom promotion” (Hearst and Chen, 2004). Indeed, despite large prevention campaigns, condom use remains very low, especially in regular relationships (Meekers et al., 2003; Glasman and Albarracín, 2003; de Walque, 2009). The low use of condoms in long-term relationship combined with the high frequency of concurrency is particularly worrisome and implies that the majority of new infections in Africa occur in regular relationships (Halperin and Epstein, 2004; Mah and Halperin, 2010). The aim to this paper is to explain the low level of condom use by a fatalistic response following an unprotected intercourse. Fatalism may indeed emerge because people exaggerate the risk of HIV transmission following an unprotected intercourse. In the light of the proposed theory, this paper discusses alternative policies for reducing risk-taking behavior due to fatalism.

When non-specialists are asked about the probability of getting infected with HIV during an unprotected sexual relationship with an infected individual, their answer is usually sharply biased. Data from Burundi shows that university students estimate at 81.4% on average the probability of contracting HIV during a single unprotected intercourse with a partner who is HIV-positive (table 1). In a recent survey in Malawi, Delavande and Kohler (2009) find a similar pattern: respondents expect on average a transmission rate of 86.8% during a single unprotected intercourse with someone who is infected. HIV/AIDS is hence perceived as a very prolific disease whose transmission is easy, if not mechanical. However, in reality, the transmission rate per coital act for heterosexual intercourse is generally lower.
than 1% (Boily et al., 2009). Such a misconception is probably due to the high number of infected individuals and the large media coverage of AIDS prevention campaigns. The general overestimation of the transmission rate per act of HIV is not confined to Africa, but is also found in US and Canada (Linville et al., 1993; Pinkerton and Abramson, 1997; Knauper and Kornik, 2004).

Intuitively, one may think that the overestimation of the transmission rate of HIV is favorable. At first glance, a higher expected risk should lead to greater motivations for safer behavior, and should consequently lower the incidence of the disease. This is probably why prevention campaigns neglect to inform people about the true transmission rate per act of HIV.

However, a more subtle and pernicious factor may counterbalance the positive effect of overestimating the transmission rate. When deciding to engage in sexual intercourse, couples should decide to use or neglect protection, knowing that condoms are effective for preventing a large proportion of HIV transmissions through sexual intercourse2 (Chimbiri, 2007). If unsafe practices were occasional and exceptional, the disease would die out in a few years, considering its low transmission rate. Although people are generally likely to use condoms during casual and commercial sexual encounters, they are rarely used in longer-term relationships; couples who engage in risky sex once generally repeat this behavior in the future (Flood, 2003; Meekers et al., 2003; Chimbiri, 2007). This may be partly due to the confidence gained in the partner or to the desire to have children (Moore and Oppong, 2007; Bravo et al., 2010).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs.</th>
<th>Total</th>
<th>Male</th>
<th>Female</th>
<th>p-value (H0: M=F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>508</td>
<td>25.2</td>
<td>25.5</td>
<td>24.7</td>
<td>0.00</td>
</tr>
<tr>
<td>Sex</td>
<td>506</td>
<td>66.8%</td>
<td>33.7</td>
<td>16.0</td>
<td>0.00</td>
</tr>
<tr>
<td>Expected transmission rate (β)</td>
<td>478</td>
<td>81.4%</td>
<td>18.6</td>
<td>81.1%</td>
<td>0.82</td>
</tr>
<tr>
<td>Expected prevalence in Burundi</td>
<td>440</td>
<td>27.9%</td>
<td>73.5</td>
<td>42.1%</td>
<td>0.00</td>
</tr>
<tr>
<td>Life expectancy with HIV</td>
<td>419</td>
<td>8.0</td>
<td>8.0</td>
<td>8.3</td>
<td>0.56</td>
</tr>
<tr>
<td>Participated in a prevention training</td>
<td>508</td>
<td>87.8%</td>
<td>86.9</td>
<td>89.3%</td>
<td>0.44</td>
</tr>
<tr>
<td>Already had sexual relationship</td>
<td>505</td>
<td>48.3%</td>
<td>51.7</td>
<td>43.7%</td>
<td>0.17</td>
</tr>
<tr>
<td>Condom at the first encounter with the last partner</td>
<td>242</td>
<td>48.3%</td>
<td>51.7</td>
<td>43.0%</td>
<td>0.31</td>
</tr>
<tr>
<td>Always used a condom with the last partner</td>
<td>228</td>
<td>21.0%</td>
<td>24.4%</td>
<td>11.7%</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1: HIV beliefs and behavior among 508 students at the University of Burundi

Another explanation, not covered in the HIV/AIDS literature, is the fatalism induced by the overestimation of the transmission rate. A sense of fatalism may rationally emerge after having been used to risk-taking behavior within a regular relationship, or after having “accumulated” a high amount of risk in the past. In this paper, these two fatalistic responses are modeled and analyzed.

The “in-relationship fatalism” typically emerges in regular relationships. Let us imagine a young adolescent, called i, who begins a romance with a partner j. If he is aware of HIV/AIDS and believes that there is a probability, even the slightest probability, that j is infected, he will probably decide to use a condom when he starts having sex with j. Consider now that at time t, i and j engage in an unplanned and unprotected sexual encounter because of the “heat of the moment”. Let us assume that i trusts the contraceptive methods and the fidelity of his partner. Then, if his estimation of the transmission rate is high, for instance 80%, the individual i may not want to use condoms anymore with j after this first unprotected encounter. Indeed, he may think: “On the one side, if j is HIV-positive, it is very likely that the virus was transmitted. If this is the case, I don’t have to care anymore about protection. On the other side, if j is HIV-negative, I don’t have to use condoms with j”. Hence, even if he does not know precisely the serostatus of j, the individual i will probably not use condoms anymore with j after the first unprotected sexual encounter. It is worth noting that this fatalistic response would not emerge if i’s estimate of the transmission rate was low. In this latter case, i would probably continue to protect himself in further sexual acts with j, as the HIV virus would not have necessarily been transmitted during the unprotected intercourse. In the extreme case where the transmission rate per sexual act is perceived to be zero, condoms are never used.

2Condoms are effective for preventing between 80 and 95 percent of all HIV transmissions through sexual intercourse (Pinkerton and Abramson, 1997; Davis and Weller, 1999; Hearst and Chen, 2004)
The “accumulation fatalism” emerges when an individual has engaged in high risk sexual activities with different partners, and if he does not know his serostatus. In this case, the individual will probably continue to engage in high risk sexual behavior because he believes that he is very likely to be already infected. The overestimation of the transmission rate accentuates this fatalistic response by amplifying the perceived likelihood of being already infected.

In summary, before the first unprotected intercourse, a positive correlation is expected between the expected transmission rate and the use of condom. Because of fatalism, after a certain amount of risk-taking, the propensity to engage in risky sex is expected to be a U-shaped function of the expected transmission rate. The aim of this paper is to construct a model representing this U-shaped relationship between the expected transmission rate and condom use, both in the case of regular and casual partners.

The perception of the transmission risk of HIV has rarely been considered in empirical research on condom use (Delavande et al., 2011). To this regard, two studies are important to notice. Meekers and Klein (2002) and Lammers et al. (2009) find a positive correlation between risk perception and condom use for casual relationships, but no significant correlation for regular partners. For both men with regular and casual partners, Meekers and Klein (2002) find that later levels of condom use are significantly lower than earlier levels. These findings support my model: a high expected transmission rate encourages protective behavior during the first sexual encounter with a partner (and therefore during casual encounters), but favors risk-taking behavior after the couple has failed to protect itself once. Because of their lack of theoretical background, these two studies fail to identify the U-shaped relationship between the expected transmission rate and condom use. Both studies do not distinguish between the expected probability that the partner is infected and the expected transmission rate of HIV.

Similarly, the complex links between HIV/AIDS knowledge and the decision to use condoms have rarely been examined from a theoretical point of view. First, in their seminal paper, Philipson and Posner (1994) construct a model comparing the private demand for information with the public provision of AIDS education. They subsequently discuss the hypothesis that providing truthful information may encourage dangerous behavior because people often exaggerate the infectivity of the AIDS virus. This latter assertion is not explicitly formalized, and their analysis does not distinguish between the perceived transmission rate and the perceived probability that the partner is infected. More recently, Tremblay and Ling (2005) set up a model of the decision to have sex and to use condoms, in the light of their perceptions of the probability of HIV transmission. Regrettably, as their model is deterministic, it does not predict the possible negative consequences of overestimating the transmission rate of HIV after an unintended and unprotected sexual encounter. Third, a related strand of the literature examines the relationship between learning HIV status and behavioral change (Boozer and Philipson, 2000; Thornton, 2008; Delavande and Kohler, 2009; De Paula et al., 2009; Gong, 2010). Theoretically, these studies predict that only people surprised by the test result should change their behavior. The direction of this change depends on the relative importance of expectation revisions and altruism. Empirical analyses lead to contradictory results. Finally, in line with the intuitions of this paper, O’Donoghue and Rabin (2001) propose a model of repeated risky choices where people have irrational beliefs about the likelihood of bad outcomes. They show that the overestimation of the riskiness of an activity may have two opposite consequences on behavior: people may reduce indulgence so as to avoid the bad outcome, or increase risk-taking if the bad outcome appears to be unavoidable.

In this paper, the model of O’Donoghue and Rabin (2001) is extended to study the links between condom use and the per-act expected transmission rate in both regular and casual relationships. I propose a dynamic stochastic model of repeated risky choices incorporating risk evaluation. The analysis considers the time dimension of the condom use decision making and distinguishes between the expected probability that the partner is infected and the expected transmission rate of HIV. The aim is to show that biased knowledge of the expected transmission rate may have a positive and a pernicious effect, yielding a U-shaped relationship between risk and the expected transmission rate. Simulations of the model then answer three unexplored questions: “Should prevention campaigns disclose the true transmission rate of the HIV/AIDS virus?”; “Which expected transmission rate would maximize condom use?” and “What would happen if a rumor or an Internet buzz propagates the rumor that the transmission rate is very low?”.

The second section develops a theoretical model of an individual who is engaged in a long-lasting relationship and who has to choose between safe and risky sex. It is shown that an upward-biased evaluation of the transmission rate
per-act may generate “in-relationship fatalism” when unplanned risky sexual encounters are probable. This section ends with simulations of the model that are calibrated by using data from Burundi. In the third section, the model is adapted to an individual engaging in casual sex with different partners. It is shown that the accumulation of risk-taking behavior may discourage the use of condoms by fatalism. In the fourth part, predictions of the model are compared to empirical evidence. The model is then used to assess the impact of policies disclosing that the transmission rate is lower. The last section discusses the findings and addresses their implications for policy and further research.

2. A stochastic model for long-lasting relationships

The aim of this section is to develop a theoretical model predicting a U-shaped relationship between condom use and the expected transmission rate of HIV in the case of a regular relationship. This model links condom use with specific parameters such as the expected transmission rate, the expected probability that the partner is infected and the expected length of life with HIV.

The intuition of the model is as follows. At the beginning of a regular relationship, an individual is expected to use condoms if he fears an HIV infection. In this case, believing that the transmission rate of HIV is high encourages condom use. In order to explain behavioral changes, it is assumed that unprotected sex can occur accidentally in the “heat of the moment”. The model shows that, after an unplanned and unprotected intercourse, partners are subject to “in-relationship fatalism” if they overestimate the transmission rate of HIV. In this case, they will believe that the virus would have anyway been transmitted during the unprotected intercourse if one of the partners was infected, thereby rendering unnecessary the use of condoms in the future.

2.1. Set-up

A healthy and risk-neutral individual \( i \) begins his sexual life at time \( t = 0 \). He has regular sexual encounters with a partner \( j \) (one sexual encounter per period). The model assumes an infinite-horizon in discrete time\(^4\). As long as he is alive, the individual \( i \) has to choose in each period whether or not to use a condom\(^5\). If protection is used at time \( t \), \( s_t = 0 \), and the sexual relationship is assumed to be risk-free. Otherwise, \( s_t = 1 \) and \( i \) expects that there is a positive probability \( \beta \) that the virus is transmitted if the partner is infected\(^6\), with \( 0 < \beta < 1 \).

The instantaneous utility of risky sex is normalized to 1, and the instantaneous utility of safe sex is denoted \( \theta \). Protected sex is assumed to be less pleasurable than unprotected sex\(^7\): \( 0 < \theta < 1 \). Besides the utility of sex, the individual gets a positive utility \( \alpha \) for each period he is alive. \( \alpha \) represents all the other pleasures that individual \( i \) enjoys. In summary, the instantaneous utility function is binary and given by:

\[
u(s_t) = \alpha + s_t + \theta(1 - s_t) (1)\]

Let us denote \( d_t \) the expected probability of being dead at time \( t \), and \( \rho \) the discount factor. The optimization program of \( i \) is then given by:

\[
\max_{s_t} U(s_t, d_t) = \sum_{t=0}^{\infty} \rho^t u(s_t)(1 - d_t) (2)\]

\(^3\)The sexes of \( i \) and \( j \) have no impact on the predictions of the model.

\(^4\)By assuming a possible infinite life, an “end of life” effect in which the individual chooses risky behavior because he knows that the optimization problem will finish soon is avoided.

\(^5\)The predictions of the model would not be affected if the decision to use condoms is taken jointly by \( i \) and \( j \).

\(^6\)In order to clarify the presentation, the expectation symbols \( \mathbb{E} \) are omitted. However, let us keep in mind that all variables are subjective for the individual \( i \).

\(^7\)See figure 1(d) below. Few people may prefer to use condoms because a sexual act lasts more when it is protected. These people are always expected to use condoms if available. For them, whether the expected transmission rate is high or low has therefore no impact on their behavior.
It is assumed that AIDS is the only cause of death\(^8\). Therefore, the expected probability of dying only depends on self-protection choices during previous sexual acts. The individual \(i\) takes rational decisions based on a binomial epidemiological model with a constant expected probability of transmission per coital act\(^9\). The expected probability that the partner \(j\) is infected is denoted \(p_j\) \((0 < p_j < 1)\). Remember that \(i\)'s expected transmission rate per coital act is denoted \(\beta\). The probability of being infected after \(n\) sexual encounters is then given by the binomial model: \(p_j \left[1 - (1 - \beta)_n\right]\). If infected, the individual \(i\) expects to die after \(\lambda\) periods. Given these parameters, the expected probability of being dead at time \(t\) is given by:

\[
d_t = p_j \left[1 - (1 - \beta)^{\text{period } t}\right] \quad (3)
\]

This deterministic model is solved in the following section. In section 2.3, this basic framework is extended to a stochastic model, which accounts for the possibility of unplanned and unprotected sexual encounters. Section 2.4 identifies the u-shaped relationship between condom use and the expected transmission rate and discusses the impact of overestimating of the transmission rate. Simulations of the model are presented in section 2.6.

### 2.2. Solution of the deterministic model

Within this deterministic framework, propositions 2.1 and 2.2 first show that \(i\) chooses either safe sex or risky sex throughout his life, but never both. Propositions 2.3 and 2.4 then show that \(i\) chooses to engage in protected sex if the expected transmission rate and the expected probability that the partner is infected are high. Indeed, when \(i\) never engaged in risky sex, he is more prone to use condoms if the HIV virus is expected to be highly contagious.

**Proposition 2.1.** If safe sex is chosen in the first period, then safe sex is chosen in all future periods.

**Proof** If \(i\) chooses to engage in safe sex in the first period, the optimization program remains the same in the second period. Indeed, a safe sexual encounter has a zero expected risk of transmission, engaging in safe sex in \(t = 0\) has no impact on the expected probability of dying, and therefore no impact on the maximization program of the next periods. \(\square\)

**Proposition 2.2.** If the individual \(i\) chooses to engage in risky sex at time \(t = 0\), condoms will never be used with \(j\).

**Proof** Intuitively, engaging in risky sex at time \(t = l\) increases the probability of being dead in \(t = l + \lambda\). This encourages risky practices ensuring short-term utility gain. The formal proof is in appendix. \(\square\)

Propositions 2.1 and 2.2 imply that individual \(i\) engages in either safe sex or risky sex throughout his life span, but in never both. The individual \(i\) engages in risky sex if the expected lifetime utility of risky sex \(R_0\) is greater than the expected lifetime utility of protected sex \(S_0\) that is, if the ratio \(\frac{R_0}{S_0}\) is greater than zero\(^{10}\). This is summarized in the following proposition.

**Proposition 2.3.** Within this deterministic framework, the individual \(i\) chooses to engage in safe sex if:

\[
P_0 = \frac{R_0}{S_0} = 1 = \frac{\frac{1 + a}{1 + P} - \frac{(1 + a)p \beta \rho}{(1 - p)(1 - p - \beta)}}{\frac{\rho a}{1 - P}} = 1 \leq 0. \quad (4)
\]

**Proof** Proof in appendix.

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\(^8\)As explained by O’Donoghue and Rabin (2001), the discount factor \(\rho\) may be interpreted as including implicitly all other factors of death.

\(^9\)In reality, the transmission rate of HIV is not constant: the infectivity is higher in the first and the later stages of the disease (Wawer et al., 2005; Boily et al., 2009). However, a constant expected transmission rate per-act \(\beta\) is assumed because individuals are not well informed about the different stages of the AIDS disease and because it is difficult to visually distinguish HIV-positive people from uninfected individuals.

\(^{10}\)\(R_0 - S_0\) is rescaled by \(S_0\) in order to obtain a dimensionless ratio which allows a interpretation in terms of percentage and facilitates comparisons.
$P_0$ represents the propensity to engage in risky sexual behavior before any unprotected intercourse between $i$ and $j$. The numerator of the fraction is the expected lifetime utility of risky sex $R_0$. The denominator represents the expected lifetime utility of safe sex $S_0$.

The inequality (4) implies that risky sex is avoided if the expected transmission rate $\beta$, the expected serostatus of partner $p_j$ and the utility of safe sex $\theta$ are sufficiently high ($\frac{\partial R_0}{\partial \beta} < 0$, $\frac{\partial R_0}{\partial p_j} < 0$, $\frac{\partial R_0}{\partial \theta} < 0$). Indeed, the fear of infection and death prevents excessive risk taking. This is the favorable direct effect of an upward-biased expected transmission rate. A higher life expectancy of HIV-positive individuals $\lambda$ encourages risky behavior, which suggests that the availability of antiretroviral therapies may discourage condom use ($\frac{\partial P_0}{\partial \lambda} > 0$). An increase in the instantaneous utility of other activities than sex $\alpha$ favors safe sex because people are less willing to jeopardize years of pleasant life for risky sex ($\frac{\partial P_0}{\partial \alpha} < 0$). Similarly, a reduction in the discount factor favors short-termist risky behavior ($\frac{\partial P_0}{\partial \rho} < 0$). This shows that improving life prospects of vulnerable people may reduce risk-taking behavior and therefore reduce the spread of the virus.

Let us examine in detail the relationship between the propensity $P_0$ and the expected transmission rate $\beta$. For the extreme case $\beta = 0$, $P_0$ is positive, maximal, and equal to $\frac{1 - \rho}{\alpha + \rho}$. Indeed, if the disease is not feared, $i$ has no reason to use condoms. At the other extreme, if $\beta = 1$, $P_0$ is equal to $\frac{1 - \rho}{\alpha + \rho} - \frac{p_j(1 + \rho)^{\rho t}}{(\alpha + \rho)^{\rho t}}$, which is negative if the expected probability that the partner $j$ is infected is higher than the threshold $\bar{p} = \frac{(1 - \rho)}{(1 + \rho)^{\rho t}}$. Between these two extreme cases, the propensity $P_0$ is a decreasing function of $\beta$. This is summarized in the following proposition.

**Proposition 2.4.** Before any unprotected sexual encounter, the propensity to engage in risky sexual behavior $P_0$ is a decreasing function of the expected transmission rate $\beta$.

**Proof** This proposition directly follows from the fact that the derivative of $P_0$ is negative: $\frac{\partial P_0}{\partial \beta} = -\frac{p_j(1 + \alpha)(1 - \rho)p^\rho t}{(\alpha + \rho)(1 - \rho)^{\rho t}} < 0$.

### 2.3. A stochastic framework

In the previous section, the decision to engage in risky behavior was assumed to be deterministic. This framework does not explain why people may change their sexual behavior and how unplanned risky encounters may occur. Let us extend the preceding framework and assume that the decision to use condom is influenced by a random process $\epsilon_t$, which is independent from the other parameters, and unknown to the decision-maker. This random process represents the “heat of the moment” or the possibility of a condom failure.

Within this stochastic framework, unintended risky encounters are possible. For example, if $P_0$ is negative, such that $i$ would choose to engage in safe sex in a deterministic framework, the individual $i$ nevertheless engages in risky sex if the outcome of the random process $\epsilon_t$ is high, such that $P_0 + \epsilon_t > 0$. In this case, the rational choice of $i$ is to practice safe sex, but the resulting behavior is to engage in risky sex.

Let us generalize this example. In line with our previous notation, let us denote $P_N$ the propensity to engage in risky sexual behavior if $i$ and $j$ have had $N$ unprotected sexual encounters before the time of the decision $t$. Then, the stochastic framework assume that the individual $i$ engages in safe sex at time $t$ if $P_N + \epsilon_t \leq 0$. Within this framework, even if the propensity $P_N$ is negative, such that $i$ would choose to engage in safe sex in a deterministic framework, the individual $i$ engages in unprotected sex if the outcome of the random process $\epsilon_t$ is high, such that $P_N + \epsilon_t > 0$. Because of the random process $\epsilon_t$, we have to distinguish people’s choices, which are determined by the propensities $P_N$, and people’s behavior, which is determined by the conjunction of the propensities and the outcomes of the random process.

The fact that unplanned risky encounters may occur is in line with empirical evidence. For example, in the sample of students at the University of Burundi, 39.3% of those who used a condom during their first sexual encounter with their regular partner admitted an irregular use of condoms. The main justifications given to explain the non-use of condoms during the last encounter are the lack of condom (41%) and the confidence in the partner (41%). It is also worth noting that 25.5% of the respondents ever experienced a condom failure. Sanders et al. (2012) compile an interesting review of frequent errors in condom use.
What functional form for $\epsilon_i$ does realistically represent an error due to the “heat of the moment” or to a condom failure? Let us consider two classes of functional forms for $\epsilon_i$: Bernoulli processes, and the class of distributions whose cumulative distribution function (CDF) is continuous and strictly increasing on $\mathbb{R}$ (for example a normal distribution). We will show that the latter class is more realistic in the case of HIV/AIDS and behavior.

Let us first consider that $\epsilon_i$ is a Bernoulli process which takes the value 0 with a probability $x$, and a high value\(^{11}\) with a probability $1 - x$. In this case, if $i$’s propensity to engage in risky behavior is negative such that the individual $i$ chooses safe sex, he nevertheless engages in risky behavior with a probability $(1 - x)$. This Bernoulli process consistently represents the probability $x$ of a condom failure, which may affect every user with the same probability. However, it does not realistically represents an error due to the “heat of the moment”. A simple example shows that this functional form may lead to inconsistencies. If $i$ starts a relationship with $j$ and believes that the transmission rate is high ($N = 0$, $\beta = 1$), he will choose to use condoms if the expected probability that the partner is infected $p_j$ is higher than the threshold $\tilde{p}^{12}$. If $i$ knows that his partner is infected ($p_j = 1$), he has the same probability to engage in risky sex than if the expected probability that $j$ is infected is as low as $\tilde{p}$. In both cases, the probability to engage in risky sex is equal to $(1 - x)$. This is inconsistent with the view that an individual is more likely to use condom when he knows that his partner is infected. The formulation of $\epsilon_i$ as Bernoulli process is therefore rejected.

Assuming that the CDF of $\epsilon_i$ is continuous and strictly increasing on $\mathbb{R}$ prevents this type of problems (any normal distribution satisfies these properties). In this case, the lower the propensity to engage in risky sex, the lower is the probability to engage in risky sex. This is much more intuitive: when the partner is known to be infected, the probability to use condoms is higher than when the partner’s expected probability to be infected is low, but above $\tilde{p}$. This class of distributions can also accommodate the case of a condom failure\(^{13}\). In what follows, the CDF of $\epsilon_i$ is therefore assumed to be continuous and strictly increasing on $\mathbb{R}$.

The following section studies how people’s behavior is altered after an unplanned and unprotected intercourse. The appendix B deals with the case where $i$ and $j$ have engaged in $N$ unplanned sexual encounters before the time of the decision.

### 2.4. Decision after an unprotected intercourse

Within this stochastic framework, let us assume that an unprotected sexual intercourse occurred in $t = -1$, like an external shock. This unprotected relationship may be due, for example, to the impulsive nature of sex or to a condom failure. Because of the risky sexual encounter which occurred at time $t = -1$, the expected probability of being alive at time $t + \lambda - 1$ is reduced.

If individual $i$ would have chosen risky sex even without this unplanned and unprotected intercourse, the optimization program is not affected and the individual will continue to choose risky sex. Conversely, if $i$ would have chosen safe sex in the absence of the unplanned intercourse, his life expectancy is reduced. His optimization program is therefore affected and $i$’s decision-making depends on his assessment of the parameters. Proposition 2.3 is modified as follows:

**Proposition 2.5.** After the unplanned risky encounter in $t = -1$, the individual chooses safe sex if:

$$P_1 = \frac{1 + \alpha}{1 - p} - \frac{(1 + \alpha)p\beta \alpha^{-1}}{1 - p} - 1 \leq -\epsilon_0. \quad (5)$$

---

\(^{11}\) A high value means a value which is higher than the absolute value of minimum of the propensities that is, higher than $|P_0(\beta = 1)| = \frac{1 - \alpha}{1 + \alpha} - \frac{1 - \alpha}{1 + \alpha}$.  

\(^{12}\) The threshold $\tilde{p}$ was introduced at the end of section 2.2. It is the probability $p_j$ for which $P_0(\beta = 1) = 0$.  

\(^{13}\) For example if the CDF is equal to or lower than the probability $x$ of a condom failure for $\epsilon_i > |P_0(\beta = 1)|$. 

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Proposition 2.6. The propensity $P_j$ to engage in risky sexual behavior after an unprotected sexual encounter is a U-shaped function of the expected transmission rate $\beta$. The transmission rate which minimizes the propensity $P_j$ is given by:

$$\beta_j^* = \frac{-(1 - \rho) + \sqrt{(1 - \rho)(1 - \rho\rho^{t+j+1})}}{\rho(1 - \rho\rho^{t+2})}. \quad (6)$$

Proof. Proof in appendix.

Propositions 2.5 and 2.6 may be easily extended to the case where $i$ and $j$ engaged in $N$ unplanned and unprotected sexual encounters before $t = 0$. The higher the number of unprotected sexual acts which occurred before the time of the decision in $t = 0$, the lower is the life expectancy of $i$ and the higher is his propensity to engage in risky sex. The proposition B.1 in appendix enlarges the scope of proposition 2.5 for the case where $i$ and $j$ have engaged in $N$ unplanned and unprotected sexual encounters before $t = 0$. The proposition B.2 in appendix proves that the propensity to engage in risky sex after $N \geq 1$ unprotected encounters is also a U-shaped function of the expected transmission rate, with a minimum $\beta^*_N$ given by:

$$\beta^*_N = \frac{-(N + 1)(1 - \rho) + \sqrt{(N + 1)^2(1 - \rho)^2 + 4\rho(1 - \rho)N(1 - \rho\rho^{t-N-1})}}{2N\rho(1 - \rho\rho^{t-N-1})}. \quad (7)$$

Proof. Proof in appendix.
Before running simulations, let us conclude the resolution of the model by describing, in words, the behavior of the individual $i$ throughout his life span. At the beginning of his sexual life, if $i$ believes that his partner is likely to be infected, he will choose to wear condoms. However, unplanned and unprotected sexual encounters may occur by negligence or because of a condom failure that is, if the outcome of the random process $\epsilon_i$ is high. In this case, the individual $i$ “switches” from $P_0$ to $P_1$; at the next period, his decision-making will be based on the propensity $P_1$ as he engaged in risky sex with his partner. If he indulges in risky sex once again, he will “switch” to $P_2$, and so on. Within this stochastic framework, the number of past risky encounters is hence endogenously determined by the propensities $P_N$ and by $\epsilon_i$. After few unprotected sexual encounters, if he overestimates the transmission rate of HIV, $i$ may be subject to fatalism and not want to use condom anymore with his partner. In reality, the probability that $i$ was infected during the reckless sexual encounters is low as the transmission rate of HIV is generally lower than 1%. Because the propensities $P_N$ for $N \geq 1$ are U-shaped functions of $\beta$, disclosing to $i$ that the transmission rate is lower may therefore encourage him to continue to use condoms.

2.5. Parameterization

The model is parameterized with a compilation of values taken from the literature, as well as estimates from a dataset collected at the University of Burundi in May 2010. The sample is constituted of 508 students who answered questions related to their sexual behavior and their knowledge about HIV/AIDS. The means and the distribution of useful variables are reported in table 1 and figures 1(a) to 1(d). The following list identifies a benchmark value for each parameter of the model, and attaches to them a range of realistic values:

- $\beta$ is the expected transmission rate per act. Because it is a probability, it ranges between 0 and 1. The benchmark value for $\beta$ is 81.4% (table 1). Figure 1(a) shows the histogram of the expected transmission rate for our sample of Burundian students.
- $p_j$ represents the expected probability that the partner is infected. Students’ estimates of HIV/AIDS prevalence in Burundi are shown in figure 1(b). Two benchmark values are considered for $p_j$: the average expected prevalence reported by students, 27.9%, and the official prevalence reported by UNAIDS in 2009, 3.3%.
- $\lambda$ is the life expectancy of a person infected with HIV (expressed as the remaining number of sexual encounters). If an average of two sexual encounters per week is assumed (Wawer et al., 2005) and knowing that a seropositive person has an average life expectancy of 10 years without ARV therapies (Oster, 2010), the benchmark value for $\lambda$ is 1,000. Students’ estimates of life expectancy of an infected individual are shown in figure 1(c).
- $\theta$ represents the relative utility of safe sex over unprotected sex. In order to find an objective benchmark value for $\theta$, several papers assessing the relative price of risky sex in commercial encounters were compared (Rao et al., 2003; Gertler et al., 2005; De la Torre et al., 2010). The premium for risky sex, $\pi$, is highly variable across countries. It ranges from 23% in Mexico (Gertler et al., 2005) to 66 – 79% in India (Rao et al., 2003). The relative utility of safe sex over unprotected sex $\theta$ is equal to: $\theta = 1/(1 + \pi)$. Hence, $\theta$ ranges from 0.55 in India to 0.81 in Mexico. The benchmark value for $\theta$ which is used in the simulation is 0.7, an intermediate value between these two extremes. Figure 1(d) shows that protected sex is less valued than risky sex among students of the University of Burundi. The category 1 corresponds to the answer “a protected encounter gives much less pleasure than a risky encounter”. The category 5 corresponds to the answer “a protected encounter gives much more pleasure than a risky encounter”.
- $\rho$ is the subjective discount factor from one period to the other. In the model, one period of time corresponds to one sexual act. There is no consensus in the literature about the mean value of the subjective discount factor. The simulations of this model rely on Botelho et al. (2006) because their experiment was undertaken in a developing country environment, and because they make the use of a “front end delay” on the early options to avoid confounding time preference with the credibility of payment. In line with their results, a yearly discount rate of 0.127 is assumed, which corresponds to a discount factor of 0.999 per period (one time period equals half a week).
- $\alpha$ represents the utility gained from other sources of pleasure than sex. The method and the data used in Kahneman et al. (2004) was used to compute a value for $\alpha$ of 67.9.
2.6. Simulations

Let us study how the behavior of people is affected by the expected transmission rate for these benchmark values of the parameters. Figures 2(a) and 2(b) show the propensities $P_N$ for $N = 0, 1, 2, 3$ and 10. In figure 2(a), the expected probability that the partner is infected is set to 27.9%, which is the average prevalence reported by students in Burundi. Figure 2(b) shows the case where $p_j$ is equal to 3.3%, the true prevalence in Burundi (UNAIDS estimate in 2009). Remember that the propensities $P_N$ determine individual’s rational choice, but that the resulting behavior also depends on the outcome of the random process $\epsilon_t$. Because the cumulative distribution function of $\epsilon_t$ is continuous and strictly increasing, the probability to engage in risky sex is strictly increasing with the propensity to engage in risky behavior. The propensity and the probability to engage in risky sex are therefore intrinsically linked concepts in this model. The further below the X-axis is the propensity to engage in risky sex, the lower is the probability to indulge in risky sex. If unprotected encounters occur, the propensity considered by the individual $i$ in his decision-making switches from $P_0$ to $P_1$, then to $P_2$, and so on.

As anticipated, the propensities to engage in risky sex are higher when the expected probability that the partner is infected is low. It is worth noting that when $p_j$ is lower than 1.37%, the propensities $P_N$ are positive for all values of $\beta$, and risky sex is always chosen.
Both figures show that the propensity $P_0$ is a strictly decreasing function of $\beta$. For each $\beta$, $P_0$ is lower than the other $P_N$'s. A feature which is striking is that $P_0$ is almost flat for $\beta > 5\%$. This suggests that disclosing that the transmission rate is somewhere between 5% and 100% would have little impact on people’s risk-taking when $N = 0$. We will come back to this argument in section 4.3 when discussing which $\beta$ should be disclosed in prevention campaigns.

In accordance with proposition 2.5 and proposition B.2, the propensities to engage in risky sex are U-shaped functions of the expected transmission rate when $N \geq 1$. When the expected transmission rate is high, for example 81.4% as for Burundian students, figures 2(a) and 2(b) show that people are inclined to engage in risky sex after few unplanned and unprotected encounters because they think that the HIV virus was anyway transmitted if their partner is infected. This suggests that the overestimation of the transmission rate may favor fatalism. Disclosing that the transmission rate is lower that 81.4% would encourage safer behavior.

For $p_j = 27.9\%$, the minimum of the propensity $P_1$ is equal to 3.4%. This means that the transmission rate which minimize risk-taking behavior is 3.4% for an individual who experienced an unplanned and unprotected encounter. In this case, this person may still want to use condoms after the unprotected encounter because he knows that the virus was not necessarily transmitted if his partner is infected.

Similarly, $\beta_1^*$ is equal to 2.4%, $\beta_2^*$ to 2% and $\beta_{10}^*$ to 1%. Figure 3(a) shows the $\beta_N^*$'s as a function of the number of past risky encounters $N$. For each $N$, it gives the expected transmission rate which minimizes the probability to engage in risky behavior.

Figure 3(b) shows how $\beta_1^*$ is affected by the discount factor $\rho$ and the expected probability that the partner is infected $p_j$. What is striking is that the $\beta_N^*$'s are little affected by changes in $p_j$. For $p_j = 3.3\%$, $\beta_1^*$ is equal to 3.3%, $\beta_2^*$ to 2.3%, $\beta_3^*$ to 1.9% and $\beta_{10}^*$ to 1%. In fact, when $\rho$ is high, a good approximation of formula (7) which defines the $\beta_N^*$'s is $\beta_N^* \approx \sqrt{\frac{1-p_j}{N}}$, which only depends on $\rho$ and $N$. Figure 3(a) is therefore valid for any value of $p_j$.

The impact of $\rho$ on the $\beta_N^*$’s is more significant. If the yearly discount rate is doubled, $\beta_1^*$ increases to 4.5% ($\rho = 0.9978$). Similarly, if the time period between two sexual acts is doubled (one encounter per week), $\beta_1^*$ increases to 4.6% ($\rho = 0.9977$). It is worth noting that the propensities to engage in risky sex $P_N$ are positive for all $\beta$ if $\rho < 0.995$. In what follows, we will therefore consider a range of values for the $\beta_N^*$’s, depending on whether the discount factor $\rho$ is high or low.

This section studied the emergence of fatalism for an individual $i$ engaged in a regular relationship. The propensity to engage in risky behavior $P_N$ was shown to be a U-shaped function of the expected transmission rate $\beta$ when $N \geq 1$. In the following section, this model is extended for a person who engages in casual sex with different partners. Simulations of both cases are confronted to empirical evidence in section 4.
3. A stochastic model for casual relationships

In this section, the model is slightly modified in order to describe the decision-making of an individual $i$ who engages in repeated casual relationship with different partners. This modified framework also predicts the emergence of fatalism. However, the U-shaped relationship between condom use and the expected transmission rate of HIV is more tenuous, and emerges only after the accumulation of several unprotected sexual encounters, especially if the expected probability that the partners are infected is low.

3.1. Modifications of the set-up

As in section 2, a healthy and risk-neutral individual $i$ begins his sexual life at time $t = 0$. In this section, $i$ meets a new partner each period. The expected probability that the new partner at time $t$ is infected is denoted $p_t$. It is assumed that $p_t$ is similar for all partners and equal to $p$.

Within this framework, the probability to get infected is slightly modified. At each period, $i$ expects a probability $p$ to meet an infected partner, and a probability $\beta$ to get infected if he engages in risky sex and if he was not yet infected. The probability to be dead at time $t$ is then given by:

$$d^*_t = \left[1 - (1 - \beta p)^{\sum_{k=1}^{t} \lambda_k}\right]$$

(8)

3.2. Decision before any unprotected intercourse

When the individual $i$ engages in casual sex with different partners, proposition 2.3 is modified as follows:

**Proposition 3.1.** If the individual $i$ engages in casual sex throughout his life, he chooses to use condoms in $t = 0$ if:

$$P^*_0 = R^*_0 \frac{\epsilon}{S^*_0} - 1 = \frac{1+\alpha}{1-p} - \frac{(1+\alpha)\beta p^*}{(1-\rho)\beta p^*(1-\beta p)} - 1 \leq -\epsilon_0.$$  

(9)

**Proof** Proof in appendix.

The main difference between inequalities (9) and equation (4) is the presence of the probability $p$ in the denominator of the second term of $R^*_0$. This new factor $p$ sharply reduces the incentive to engage in risk-taking behavior. Indeed, the risk to get infected when engaging in casual sex is higher than within a long-lasting relationship. Within a long-lasting relationship, the probability to get infected is bounded by the probability $p_j$ that the partner is infected. On the contrary, in casual sex, the individual $i$ draws a new partner at each period and this partner has a probability $p$ to be infected. In the long-run, when the individual engages unprotected casual encounters, the probability to get infected rapidly accumulates, and is therefore much higher than in a regular relationship.
The direction of the relationships between the propensity $P_0^c$ and the different parameters is not affected by the fact that $i$ engages in casual sex. Risky sex is avoided if protected sex is enjoyed and if the fear of infection is high ($\frac{\partial P_0^c}{\partial N} < 0$, $\frac{\partial P_0^c}{\partial ρ} < 0$, $\frac{\partial P_0^c}{\partial ρ} < 0$). Similarly, condoms are used if the life expectancy with HIV is short, which suggests that antiretroviral therapies may increase risk-taking behavior ($\frac{\partial P_0^c}{\partial ρ} > 0$). Similarly, an increase in the discount factor or in the instantaneous utility of other activities than sex favors safe sex because people are less willing to jeopardize years of pleasant life for risky sex ($\frac{\partial P_0^c}{\partial γ} > 0$, $\frac{\partial P_0^c}{\partial γ} < 0$, $\frac{\partial P_0^c}{\partial γ} < 0$). The following proposition formalizes the relationship between the expected transmission rate and behavior when $N = 0$.

**Proposition 3.2.** Before any unprotected sexual encounter, the propensity to engage in risky sexual behavior $P_0^c$ is a decreasing function of the expected transmission rate $β$.

**Proof** This proposition directly follows from the fact that the derivative of $P_0^c$ is negative: $\frac{\partial P_0^c}{\partial β} = -\frac{ρ(1+γ)(1-ρ)p^2}{(1-ρ)(1-ρ)p^2} < 0$.

As within long-lasting relationships, when $N = 0$, the transmission rate which minimizes the probability to engage in risky behavior is $β_0^c = 1$.

### 3.3. Decision after an unprotected intercourse

After a first unprotected sexual encounter, the propensity to engage in risky sex increases because of fatalism: the individual $i$ may indulge in risky behavior if he believes that he is likely to have been infected. This fatalistic attitude is especially strong if $β$ and $p$ are high. After a first risky encounter, proposition 3.1 is modified as follows:

**Proposition 3.3.** After an unplanned risky encounter in $t = -1$, the individual $i$ chooses safe sex if:

$$P_1^c = \frac{1+γ}{1-p} \frac{(1+γ)p^{i+1}}{p^{i+1}} - \frac{θ(1+γ)p^{i+1}}{(1-p)^{i+1}} \leq -ε_0. \quad (10)$$

**Proof** Proof in appendix.

Except for the expected transmission rate $β$, the direction of the relationships between the propensity $P_1^c$ and the different parameters is unchanged when $i$ had risky sex with a casual partner in $t = -1$.

By contrast, the relationship between the propensity to engage in risky sex and the expected transmission rate $β$ may be altered after an unplanned and unprotected sexual encounter. In section 2, we have seen that the propensity $P_1^c$ is a U-shaped function of $β$. When the individual $i$ engage in casual sex, a U-shaped relationship may also emerge, but it requires few conditions: an individual who overestimates the transmission rate will be fatalistic if he think that his partner was likely to be infected ($p$ high) or he had unprotected sex with many partners ($N$ high). In this case, a U-shaped relationship between the propensity to engage in risky sex and the expected transmission rate emerges and a decrease in $β$ may reduce the strength of the fatalistic reaction. This reasoning is summarized in the following proposition for $N = 1$.

**Proposition 3.4.** The propensity $P_1^c$ to engage in risky sexual behavior after an unprotected sexual encounter is a U-shaped function of the expected transmission rate $β$ if:

$$p > \frac{-(1-ρ) + \sqrt{(1-ρ)(1-ρ^{i+1})}}{ρ(1-ρ^{i+2})}. \quad (11)$$

In this case, the transmission rate which minimizes the propensity $P_1^c$ is given by:

$$β_1^c = \frac{-(1-ρ) + \sqrt{(1-ρ)(1-ρ^{i+1})}}{ρp(1-ρ^{i+2})} \leq β_1^c \quad (12)$$

If the condition 11 is not satisfied, the propensity $P_1^c$ is a strictly decreasing function of $β$ with a minimum in $β = 1$. 

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Proof of Theorem 1.1

This analysis may be easily extended for \( N \geq 1 \). Proposition B.3 in appendix defines the propensities \( P_{c}^* \) and proposition B.4 shows that a U-shaped relationship emerges if the following condition is satisfied:

\[
p > -\frac{(N + 1)(1 - \rho)+\sqrt{(N + 1)^2(1 - \rho)^2 + 4p(1 - \rho)N(1 - \rho^{1-N-1})}}{2Np(1 - \rho^{1-N-1})}
\]

In this case, the expected transmission rate which minimizes the propensity to engage in risky sex is given by \( \beta_{c}^* = \frac{\beta_{c}}{p} \). Conditions (11) and (13) state that fatalistic behavior emerges if \( p \) or \( N \) are sufficiently high that is, if \( i \) believes that he is likely to be HIV-positive.

Before running simulations, let us conclude this analysis by describing, in words, the behavior of an individual \( i \) who engages in casual sex throughout his life span and who overestimates the transmission rate of HIV. If \( i \) believes that his partners are likely to be infected, he will choose to wear condoms. However, unplanned and unprotected sexual encounters may occur by negligence or because of a condom failure that is, if \( \epsilon_i \) is high. In this case, the individual \( i \) “switches” from \( P_c^0 \) to \( P_c^1 \). If he indulges in risky sex once again, he will “switch” to \( P_c^2 \), and so on. After few unprotected acts, \( i \) may still want to use condoms if he is not sure to be infected. However, after a dozen of reckless sexual encounters, \( i \) may be prone to fatalism if he believes that he is likely to be infected. This fatalistic reaction is amplified if the expected transmission rate and the expected probability that the partners are infected are high. In this case, disclosing to \( i \) that the transmission rate is lower may encourage him to continue to use condoms.

3.4. Simulations

In this section, \( P_c^N \) and \( \beta_{c}^* \) are simulated in order to identify patterns of behavior of people who engage in casual sex with multiple partners.

Figures 4(a) and 4(b) shows the propensities \( P_c^N \) for \( N = 0, 1, 2, 3, 10 \) and 50. In figure 2(a), the expected probability that the partner is infected is set to the average prevalence reported by students in Burundi that is, \( p = 27.9\% \). In figure 2(b), \( p \) is equal to the true prevalence in Burundi, 3.3\% (UNAIDS estimate in 2009). The propensities to engage in risky sex are negative, except for very low values of \( \beta \) and for high values of \( p \) and \( N \). Simulations therefore predict people’s general willingness to use condoms for casual sex at the beginning of their sexual life. However, because of the random process \( \epsilon_i \), remember that risky sex is likely to occur even if people rationally prefer to engage in protected sex. The further below the X-axis is the propensity to engage in risky sex, the lower is the probability to indulge in risky sex. If unprotected encounters occur, the propensity considered by the individual \( i \) in his decision-making switches from \( P_c^0 \) to \( P_c^1 \), then to \( P_c^2 \), and so on.

![Figure 4: Propensities \( P_c^N \) as a function of \( \beta \)](image-url)

\(^{14}\)The propensity \( P_c^0 \) is positive for all values of \( \beta \) and risky sex is always chosen if \( p \) is as low as 0.0016\%.
Both figures display that the propensity \(P_c\) is strictly decreasing in \(\beta\). On the left-hand side figure, when \(p\) is high, it is striking that \(P_0\) is almost flat for \(\beta > 10\%\). When \(p\) is intermediate, \(P_0\) is flat for \(\beta > 30\%\) (figure 4(b)). When \(p\) is low, for example 0.2\% as the prevalence in United Kingdom (UNAIDS estimate in 2009), the propensity \(P_0\) increases dramatically when \(\beta\) is reduced. This suggests that disclosing that the transmission rate is somewhere between 10-30\% and 100\% would have little impact on people’s risk-taking when \(N = 0\) and \(p\) is high or intermediate, but may have a significant impact when \(p\) is low. We will come back to that in section 4.3 when discussing which \(\beta\) should be disclosed in prevention campaigns.

In accordance with proposition 3.3 and proposition B.4, the propensities to engage in risky sex are U-shaped functions of the expected transmission rate when \(N\) and \(p\) are high, but this relationship is less clear-cut than in the case of long-lasting relationships. For \(p = 27.9\%\), \(P_1\) is slightly U-shaped, with a minimum equal to 14.1\%. In this case, the transmission rate which minimizes risk-taking behavior is 14.1\%. Similarly, \(\beta_1^{**}\) is equal to 10\%, \(\beta_2^{**}\) to 8.1\%, \(\beta_{10}^{*}\) to 4.3\% and \(\beta_{20}^{*}\) to 1.8\%. Figure 5(a) displays the \(\beta_N^{*}\)’s as a function of the number of past risky sexual encounters \(N\). For each \(N\), it gives the expected transmission rate which minimizes the probability to engage in risky behavior.

When \(p\) is intermediate (\(p = 3.3\%\)), \(P_1\) is not U-shaped, but strictly decreasing in \(\beta\). In this case, the expected transmission rate which maximizes condom use after an unplanned risky encounter is \(\beta_1^{**} = 100\%\). Indeed, when the likelihood that the partner is infected is low, people know that the chance to be infected after one error is low, and therefore continue to use condoms. After two unprotected sexual encounters, the propensity \(P_1\) becomes U-shaped, with a minimum in \(\beta_1^{**} = 84.2\%\). Similarly, \(\beta_1^{*}\) is equal to 68.5\%, \(\beta_{10}^{*}\) to 36.7\% and \(\beta_{20}^{*}\) to 15.3\%.

When \(p\) is low, the number of past risky encounters \(N\) which are necessary to yield a U-shaped increases sharply. For example, when \(p = 0.6\%\) as the US prevalence of HIV, \(P_1\) is a U-shaped function of \(\beta\) if \(N\) is higher than 37. When \(p = 0.2\%\) as the prevalence in United Kingdom, the threshold is \(N \geq 249\).

Figure 5(b) shows how \(\beta_1^{**}\) is slightly affected by the discount factor \(\rho\) and strongly affected by the expected probability that the partner is infected \(p\). In what follows, we will therefore consider a range of values for the \(\beta_N^{**}\)’s, depending on whether the discount factor \(\rho\) is high or low. Different scenarios in terms of the expected probability that the partner is infected \(p\) will be discussed.

4. Empirical evidence and policy

4.1. Patterns of behavior

Before comparing the predictions of the model with empirical evidence, let us summarize and compare the two types of fatalistic reactions which were characterized in sections 2 and 3, and give an overall view of how people are expected to behave given their sexual histories and their expectations about the parameters.
At the beginning of sexual life, according to the model, people are willing to wear condoms in regular relationships if their partner is possibly infected ($p_j > 1.37\%$). By contrast, with casual partners, people always have the intention to use condoms at the beginning of their sexual life, as long as there is a probability, even the slightest probability, that the partner is infected ($p > 0.0016\%$). Despite these good intentions, errors may occur because of the “heat of the moment”, because of a condom failure or because partners want to have a baby. After few reckless sexual encounters, a sense of fatalism may discourage the use of condoms both in regular and casual relationships.

On the one side, fatalism may emerge after having been “used” to risk-taking behavior with a regular partner. This “in-relationship fatalism” is directly caused by the overestimation of the transmission rate of HIV: after one unprotected sexual encounter, partners believe they have the same serostatus and therefore conclude that condoms are not useful anymore as a prophylaxis.

On the other side, fatalism may emerge after having “accumulated” a high amount of risk in the past. In this case, the selfish individual engages in high risk sexual behavior because he believes that he is very likely to be already infected. This “accumulation fatalism” emerges after numerous unprotected acts with different partners which are at high risk of infection. The overestimation of the transmission rate accentuates this fatalistic response by amplifying the perceived likelihood of being infected.

In reality, people’s sexual histories are a mixture of regular and casual relationships. With regular partners, “in-relationship fatalism” is likely to emerge after few unplanned risky sexual encounters. After several unprotected sexual acts with different high-risk partners, the “accumulation fatalism” is likely to discourage the use of condoms, regardless of whether these relationships are casual or regular. For example, in Sub-Saharan Africa, most transmissions occur within the marriage (Dunkle et al., 2008), largely because of the low use of condoms and the high prevalence of concurrent long-term relationships (Halperin and Epstein, 2004; Mah and Halperin, 2010; Moore and Oppong, 2007). In this case, the “in-relationship fatalism” emerges with all concurrent regular partners, and all partners within a concurrency network are susceptible to become infected in the long-run. The “accumulation fatalism” may also appear, especially for people who have had many partners, or for people who believe that their regular partner has had many partners.

While “in-relationship fatalism” is self-limiting as the propensities to engage in risky sex $P_N$ are decreasing with $p_j$, it is worth noting that the “accumulation fatalism” may be self-reinforcing, especially if the transmission rate is overestimated and if reckless sex is frequent. A careful comparison between figures 4(a) and 4(b) illustrates this point. When the transmission rate of HIV is overestimated, the propensity to engage in risky sex $P_N$ is lower when the expected probability that the partners are infected $p$ is high. On the contrary, all the propensities $P_N$ for $N \geq 1$ are higher when the probability that partners are infected $p$ is high. Indeed, when both $p$ and $\beta$ are high, the decision-maker believes that he is likely to have been infected, and will be less careful in the future. In sum, a high expected probability that partners are infected $p$ foster risky behavior when errors are likely. The low use of condoms favors the spread of HIV, which in turn increases the expected probability that partners are infected. This vicious circle may partly explain the low level of condom use among high-risk populations, and especially in Sub-Saharan Africa. It also shows that exaggerating the extent of the HIV epidemics in order to decrease risk-taking may be misleading and go against the objective of reducing the spread of HIV.

The two types of fatalism are affected differently by altruism and by the availability of testing for HIV. Theoretically, only people surprised by the HIV test result should change their behavior (Boozer and Philipson, 2000). The direction of this change depends on the relative importance of expectation revisions and altruism. Empirical analyses lead to contradictory results. On the one side, Thornton (2008), De Paula et al. (2009) and Delavande and Kohler (2009) show that altruism encourages HIV-positive people to buy more condoms and to reduce extramarital affairs. On the other side, Gong (2010) finds a six-fold increase in other sexually transmitted diseases after a positive test, suggesting that selfish motivations and self-protecting behavior are more important than altruistic considerations after receiving the test result.

According to the present model, the “in-relationship fatalism” vanishes only if partners learn they are serodiscordant. In this case, they may be willing to engage in safe sexual behavior in order to protect the uninfected partner. By
contrast, if the transmission rate of HIV is overestimated and if partners already engaged in risky sexual behaviors, partners will continue to engage in unprotected sex if partners learn they have the same serostatus, if only one partner receives the test result or if partners do not disclose the result of the tests. In these cases, because the expected transmission rate is high, partners believe they have the same serostatus and “in-relationship fatalism” discourage them to use condoms.

After many risky sexual encounters with different partners, we have seen that “accumulated fatalism” may emerge if past partners are expected to be at high risk of infection and if the transmission rate of HIV is expected to be high. In this case, if an individual subject to fatalism receives a negative test result, he will conclude that the risky encounters he had were not as risky as expected. His number of past risk sexual encounters should therefore be reset to zero, which thereby wipe out “accumulation fatalism”. By contrast, if he receives a positive test result, his reaction depends on altruism: he will use condoms if he is willing to protect his new partners.

Even without having been tested, altruism is expected to mitigate “accumulation fatalism”. Indeed, an altruist person who had sex with high-risk partners may be willing to use condoms in order to protect his future partners. By contrast, altruism is not expected to attenuate “in-relationship fatalism”: as both partners expect to have the same serostatus after the first unprotected intercourse, they have no reason to be protective, even if they are altruistic.

In sum, the impact of altruism and testing on fatalism depends on the type of relationship and on whether the test results are shared with the partners. Testing seems a powerful strategy to reduce fatalistic risky behavior, especially if regular partners get tested jointly and are encouraged to share the test results, and if persons with multiple partners are altruistic. Unfortunately, existing papers studying the impact of testing on behavior do not distinguish between regular and casual partners, nor do they distinguish who is tested and whether the test results are disclosed to the partners or not. The predictions of the model can therefore not be compared to their empirical findings.

4.2. Empirical evidence

The purpose of the survey undertaken in Burundi was to highlight the general overestimation of the transmission rate per act of HIV, and also to gather information on sexual behavior of students in order to study empirically the link between the transmission rate and the use of condoms. The first objective is fulfilled, as the data show that respondents’ average estimate of the transmission rate per act is 81.4%. Unfortunately for the second objective, the transmission rate is so largely overestimated that there is no comparison group who believes that the transmission rate of HIV is low. Given the limitation of available data, a direct measurement of the U-shaped relationship between condom use and the expected transmission rate is not possible. Rather, this section shows that many predictions of the model are consistent with the existing literature.

The empirical literature which explains the use of condoms confirms that condoms are well accepted with casual partners, but not in long-lasting relationships. Using DHS data from five African countries, de Walque (2009) shows that the use of a condom at the last intercourse with spouse ranges between 1.9% and 19.8% for married couples. In comparison, for non-marital sex, the use of a condom at the last intercourse is much higher, and ranges between 23.5% and 67.9%. Similar trends were also observed in Argentina, Cameroon, US, both in homosexual and heterosexual relationships (Glasman and Albarracin, 2003; Meekers et al., 2003; Doherty et al., 2009; Brady et al., 2009). This pattern of behavior can be easily explain by the “in-relationship” fatalism and by the fact that for a given expected transmission rate, the propensities for casual sex $P_N$ are much lower than the propensities $P_N$ for long-lasting relationships.

The higher use of condoms in casual relationships is all the more striking that random episodes of reckless sex seems more prevalent with non regular partners. The correlation between intentions to engage in safe sex and condom use was shown to be higher with regular partners than with casual partners (Sheeran and Orbell, 2011; Glasman and Albarracin, 2003). Similarly, qualitative interviews in Flood (2003) show that people describe the “heat of the moment” as a feature of both casual and regular sexual relations, although it is more likely in one-off sexual episodes. This suggest that $\epsilon_t$ is on average higher in casual relationships. Even if errors are more frequent with casual partners,
the lower use of condoms in regular relationships can be explained by the “in-relationship” fatalism, and by the fact that the propensities are generally lower for casual sex.

The important strand of literature based on qualitative interviews confirms that habituation lead participants to practice unsafe sex with their regular partnerships (Rhodes, 1997). Many respondents say they abandoned condom use because they trust their partner. As an explanation for this, Flood (2003) shows that sexual practice alone can produce “trust”. During a qualitative interview a respondent told of “a casual sexual involvement in which having had intercourse once, he and his partner then didn’t worry about condoms for further episodes of intercourse that night” (Flood, 2003). This habituation is even more present in couples, with many respondents agreeing that condoms should not be used in a steady relationship (Moore and Oppong, 2007).

Interestingly, Brady et al. (2009) shows that the “in-relationship fatalism” may vanish if one of the partner believes that his main partner has been unfaithful. Their analysis shows that the perception that one’s main partner had been unfaithful, but not one’s own sexual concurrency is associated with consistent condom use and fewer acts of unprotected sexual intercourse. This self-protective reaction can be easily accommodated in the model. First, learning that the partner is unfaithful increases the expected probability that he is infected $p_j$, which lowers the propensity to engage in risky behavior. Second, concurrency implies that the two regular partners do not have necessarily the same serostatus, as the unfaithful partner may have been infected outside the regular relationship. As the “in-relationship fatalism” is based on the belief that both partners have the same serostatus, we conclude that concurrency hinders this form of fatalism.

By contrast, sexually concurrent youths who engaged in inconsistent condom use with other partners were more likely to engage in inconsistent condom use and a greater number of unprotected sexual intercourse acts with main partners Brady et al. (2009). This empirical observation is in accordance the model, and corresponds to the “accumulation fatalism”.

Finally, the empirical literature confirms the predictions of the model about the relationship between condom use and the other variables of the model. The empirical analysis of Lazarus et al. (2009) supports that condom use is positively associated with prevalence among 15-year-old adolescents in the European Union. This positive relationship between prevalence and safe sex is predicted by our model in the case of long-lasting relationships, and in the case of casual relationships when the expected probability that the partners are infected $p$ and the number of past unprotected sexual acts $N$ are low. These assumptions are likely to be satisfied for most of the 15-year-old adolescents in the sample. In accordance with our model, Rhodes (1997) and Maticka-Tyndale and Kyeremeh (2010) show that a short life expectancy is associated with higher risk-taking with respect to HIV. Finally, several authors showed that the availability of antiretroviral therapies increases risky behavior (Crepaz et al., 2004; Mechoulan, 2007; Lakdawalla et al., 2006).

Let us conclude this empirical section by discussing how to further assess the validity the theory proposed in this paper. Given the nature of this topic, organizing a random experiment which would disclose different transmission rates to different sub-populations would raise ethical concerns as it is not known how people will react to the information. It would be morally untenable to engage some people in a prevention campaign which may be harmful for them. In order to test the model, it is therefore necessary to find two very similar groups of people who differ only in the fact that they have different perceptions of the transmission rate. Random samples of the population do not fit this characterization because nearly all individuals expect the transmission rate to be very high. Hence, no comparison group is available. A quasi-experiment among medical students may solve the problem of lack of variability. By comparing the behavior of students who have already followed a course disclosing the true transmission rate with that of younger students, it should be possible to measure the impact of the disclosure on behavior. Although this identification strategy seems unbiased, its external validity may be questionable.

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15If $p$ and $N$ are high, remember that a vicious circle may emerge and amplify the “accumulation fatalism”.

18
4.3. What would happen if a prevention campaign or a rumor spreads that the transmission rate of HIV is low?

First, it is worth noting that there is no exact estimate of the true transmission rate per sexual act. In a recent meta-review of 43 publications, Boily et al. (2009) compare the male-to-female and the female-to-male HIV transmission rate per contact for low and high-income countries. Per-act estimates of selected publications range from zero to 8.2%, showing the high heterogeneity of estimates across different surveys. Boily et al. (2009) explain these large differences between developing and developed countries by the higher prevalence in developing countries of other sexually transmitted diseases and genital ulcers (GUD), and by the higher exposure to commercial sex. They calculate that genital ulcers multiply the transmission rate by 5.3, exposure to commercial sex is associated with an 11-time increase in infectivity, and the transmission rate is multiplied by 9.2 in the earlier phase of the disease, and by 7.3 in the later phase.

This clarification is all the more important since the impact of a rumor would sharply depend on which transmission rate is disclosed. Figure 6(a) shows both the propensities \( P_N \) and \( P'_N \) when the expected probability that partners are infected is high. If the transmission rate which is disclosed is higher than 5%, the impact on condom use is expected to be positive because both types of fatalism are reduced. We will come back to this in the next section when discussing which transmission rate would maximize condom use.

By contrast, in the extreme case where the rumors spreads that the transmission rate is as low as 0.04 or 0.08%, which are the pooled female-to-male and male-to-female transmission rate estimates reported by Boily et al. (2009) in high-income countries, the impact on condom use would be dramatic, especially in population where condoms are not used as a contraceptive. For the benchmark values of the parameters, a switch from \( \beta = 81.4\% \) to \( \beta = 0.08\% \) would reduce the propensity \( P_0 \) from \(-0.08\) to \(-0.03\), and reduce the propensity \( P'_0\) from \(-0.31\) to \(-0.04\). A rumor spreading that the transmission rate of HIV is very low would therefore have a strong negative impact on condom use, especially in casual relationships.

![Figure 6: Comparing casual and long-lasting relationships](image)

4.4. Which expected transmission rate would maximize condom use?

From an epidemiological point of view, it is interesting to know how to maximize the probability to engage in protected sex. This section uses simulations of the model in order to calculate a raw estimate of the expected transmission rate which maximizes condom use with high risk partners. Results of this section should be taken with caution. No empirical evidence exists on the exact distribution of sexual histories and of \( \epsilon_t \) in different populations. The knowledge of these distributions is a prerequisite to obtain an exact estimate of the transmission rate which maximizes condom use. More empirical knowledge is therefore needed to refine the estimates that are proposed. This section is nevertheless important as it is the first attempt to estimate the “safer” expected transmission rate, and to discuss how policy-makers could implement the recommendations in prevention campaigns.

Let us first focus on regular relationships. Section 2 showed that the propensity to engage in risky behavior \( P_0 \) is a decreasing function of \( \beta \), which means that the transmission rate which minimizes \( P_0 \) is \( \beta'_0 = 100\% \) (figure
6(a)). The propensity \( P_0 \) is almost flat for \( \beta \geq 5\% \), which suggests that people would not change their behavior at the beginning of a regular relationship if they believe that the transmission rate is 5\% rather than 81.4\%. After an unplanned and unprotected encounter, the transmission rate which minimizes \( P_1 \) ranges between 3.4\% and 4.6\%, depending on whether the discount rate is high or not. The ranges of \( \beta_N^* \) which minimizes the propensities are shown on figure 6(b). This figure, together with the fact that the propensity \( P_N \) is flat for \( \beta \) higher than 5\%, suggests that regular partners would be more willing to use condoms in the long-run if the expected transmission rate was around 5\%. As showed in section 2.6, the analysis for regular relationship does not depend much on the expected probability that the partner is infected.

Until now, the threshold of 5\% was estimated visually. Let us refine this estimate with simulations. Starting from \( \beta = 100\% \), figure 6(a) shows that \( P_0 \) increases and \( P_1 \) decreases when \( \beta \) is reduced. The \( \beta^* \) which equalizes the marginal increase of \( P_0, -\frac{\partial P_0}{\partial \beta} \), with the marginal decrease of \( P_1, \frac{\partial P_1}{\partial \beta} \), is equal to 4.8\% for our benchmark values of the parameters, and equal to 6.6\% when the discount factor is low (one sexual encounter per week). This \( \beta^* \) is the one which favors condom use when \( N \geq 1 \) without compromising too much condom use when \( N = 0 \). The range \([4.8, 6.6]\)% is therefore defined as the interval which maximizes condom use in regular partnership when unplanned and unprotected encounters are likely to occur.

The analysis for people who engage in casual relationships is more complex because the \( \beta_N^* \)’s which minimize the propensities to engage in risky sex strongly depend on the expected probability that partners are infected \( p \). On the one side, when \( p \) is high (\( p = 27.9\% \)), section 3.4 showed that the propensity \( P_1 \) is decreasing in \( \beta \), but almost flat for \( \beta \geq 10\% \). The propensities \( P_N \) are U-shaped as long as \( N \geq 1 \). When \( p \) is high, figure 6(b) shows that the \( \beta_N^* \)’s which minimizes the propensities to engage in risky sex in casual relationships are slightly higher than the \( \beta_N^* \)’s for regular sex. As for the regular relationship case, let us refine the estimate of the 10\% threshold. The \( \beta^* \) which equalizes the marginal increase of \( P_0^* \) with the marginal decrease of \( P_1^* \) is equal to 20\% for our benchmark values of the parameters, and equal to 24.7\% when the discount factor is low. The range \([20, 24.7]\)% is therefore defined as the interval which maximizes condom use in regular partnership when unplanned and unprotected encounters are likely to occur. At this stage, it is important to note that the probability of being infected assigned to a casual partner or a sex worker is likely to be higher than for regular partners, thereby justifying the relevance of the analysis for \( p = 27.9\% \) in the case of casual encounters.

On the other side, when \( p \) is intermediate (\( p = 3.3\% \)), section 3.4 showed that the propensities \( P_N^* \) are decreasing in \( \beta \) for low value of \( N \) but almost flat for \( \beta \geq 30\% \). When \( N \) is high, the propensities become slightly U-shaped. Figure 6(b) shows that the \( \beta_N^* \) which minimizes the propensities \( P_N^* \) are high when \( p \) is equal to 3.3\%, especially if the discount factor \( \rho \) is low. When \( p \) is low, the propensities to engage in risky sex are always decreasing for realistic values of \( N \). Therefore, when casual partners are expected to be at low risk of infection, the expected transmission rate which maximizes condom use is high, somewhere between 30\% and 100\%.

In reality, people have different sexual histories and different kinds of relationship. For a given population, the transmission rate which would maximize condom use is therefore a combination of the three ranges of values defined hereinafore. This combination would not only depend on the past sexual histories and on the types of relationships of each person in the population, but also on the random process which affects their decision-making and on their expectation about the probability that their partners are infected. Furthermore, from the point of view of public health, what matters is not to maximize the general level of condom use, but better to maximize the level of condom use with partners at high risk of infection.

For example, in Sub-Saharan Africa, most of the HIV transmissions occur within the marriage (Dunkle et al., 2008). While the number of sexual partners reported in Africa is similar to that found in other regions, concurrent long-term relationships are more frequent in Sub-Saharan Africa (Halperin and Epstein, 2004; Mah and Halperin, 2010). The high level of concurrency partly explains the high prevalence of HIV in this region. As a consequence, the expected transmission rate which would minimize the incidence of HIV in Sub-Saharan Africa should be somewhere between 4.8\% and 24.7\%. By contrast, in western countries where the prevalence is low and where most of the sexual transmissions of HIV occur during casual encounters, the expected transmission rate minimizing the incidence of
HIV should be somewhere between 30% and 100%. These estimates are imprecise, but they give a first idea of which expected transmission rate would maximize condom use in different populations.

The question which follows naturally is how to put this in practice. For example, in a high-risk population, simulations suggest that disclosing a transmission rate somewhere between 4.8 and 24.7% may reduce the spread of HIV. This range is much higher than the true transmission rates reported by Boily et al. (2009) for heterosexual and homosexual encounters. It may be morally difficult to promote prevention campaigns which lie to people by advertising an average transmission rate of 5, 10 or 20%. Moreover, the sustainability of such policies is questionable because well-informed people would react against the misleading campaign. Some alternatives are however morally more defensible.

Given the imprecise estimates of the true transmission rate, which range from 0 to 8.2% (Boily et al., 2009), it would be not be misleading to disclose for example that the transmission rate is 3.9%, which is the pooled estimate obtained by Boily et al. (2009) for the male-to-female transmission rate with genital ulcer in the early phase of the disease. This value is very close to the optimal transmission rate, is based on scientific evidence and would not require lying to people. Second, it would be possible to disclose the per-relationship transmission rate which is about 30% in Sub-Saharan Africa, and around 10% in the United States and Europe (Oster, 2005). Third, it is possible to give imprecise information to people, insisting on the importance of condom use even after an unprotected sexual encounter because the virus is not always transmitted. Finally, people may be encouraged to engage in safer behavior by insisting on the dangerousness of dual infections (Smith et al., 2005) and other sexually transmitted diseases (STDs). For example, prevention messages about the dangerousness of dual infections have been disseminated to homosexual men in San Francisco, and 74% reported that they have engaged in safer sex practices because they were concerned about this problem (Colfax et al., 2004).

5. Conclusion

The infectivity of the HIV/AIDS virus is generally largely overestimated. In this paper, a model was constructed in order to assess the positive and negative impacts of this biased knowledge. On the one hand, a high expected transmission rate may promote safer behavior because people want to protect themselves against a disease that is seen as very infectious. On the other hand, if the virus is expected to be very contagious, a sense of fatalism may rationally emerge after having been used to risk-taking behavior within a regular relationship, or after having “accumulated” a high amount of risk in the past.

In this paper, we have shown that the fatalistic reactions take precedence over the fear of the HIV virus if errors due to the “heat of the moment” or condom failures are frequent. Fatalism therefore provides a realistic explanation for the low use of condoms in long-lasting relationships and in high-risk populations. It may also partly explain the limited impact of condom promotion programs and conditional cash transfers programs tied to the maintaining of a negative serostatus (Foss et al., 2007; Kohler and Thornton, 2012).

The presence of fatalism is amplified by the overestimation of the transmission rate. On the contrary, a too low expected transmission rate would favor the spread of HIV if people do not fear the disease anymore. The model was simulated to estimate the expected transmission rate which maximizes condom use. This “safer” expected transmission rate was estimated to range somewhere between 4.8% and 24.7% in high-prevalence countries, and between 30% and 100% in low-prevalence countries. Revealing that the transmission rate is below these ranges would encourage risky behavior, especially for people who engage in casual sex with partners who are expected to be at low risk of infection. The analysis also showed that testing may considerably reduce fatalistic reactions, especially when partners are altruistic and get tested jointly with their partners.

The body of literature studying the overestimation of the transmission rate of HIV is at an early stage. In this paper, a theoretical model was built to explain the low level of condom use with casual and regular partner. In the future, several extensions may be added to this framework to account for complexity of human behavior and improve
the precision of the simulations. First, a complete analysis of the impact of the expected transmission rate on risk-taking behavior should not only assess the impact on condom use, but also the impact on the number of partners, on the desire to have babies, on the disclosure of test results, and so on. Second, this micro-level analysis can be extended to the macro level, for example by modeling explicitly the spread of HIV given the sexual history of a population, by looking at general equilibrium effects (Greenwood et al., 2010), or by accounting for the formation of sexual networks (Pongou and Serrano, 2009). Finally, a rigorous empirical test of the predictions of the model is essential before engaging in any policy or program aiming to mitigate fatalistic responses induced by overestimation of the transmission rate of HIV.

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References


HIV/AIDS, gender, agency and empowerment issues in Africa.


Appendix A. Proofs

Proposition 2.2. If the individual \( i \) chooses to engage in risky sex at time \( t = 0 \), condoms will never be used with \( j \).

Proof Because of proposition 2.1, an individual who chooses unprotected sex at least once will not use protection in the first period. Indeed, from proposition 2.1, we know that if \( i \) chooses to use a condom in the first period, then he will always use condoms in further sexual acts. We have to prove that the choice \( s_k = 1 \) \( \forall k \leq l \) implies the choice \( s_{l+1} = 1 \), given a similar pattern of future behavior.

When the individual has to take a decision at \( t = 0 \), he has to forecast his decision at time \( t = l \) \( \forall l > 0 \). If he expects to choose risky sex from \( t = 0 \) to \( t = l - 1 \), then the utility gain from choosing risky sex in \( t = l \) is given by:

\[
\Delta U = \rho^l \left[ 1 - \theta - \sum_{t=0}^{\infty} \left[ \alpha + s_t + \theta(1 - s_t) \right] \rho^{-t} \beta^t \right],
\]

The term \( 1 - \theta \) represents the utility gain from choosing unprotected relationship at time \( t = l \). The negative sum of terms accounts for the utility loss due to the higher probability of dying in the future. If \( \frac{\Delta U}{\Delta S} \) is positive, \( i \) chooses to engage in unprotected sex.

To complete the proof, we need to show that if risky sex is preferred at time \( t = l \), then risky sex is also preferred at time \( t = l + 1 \). Formally, given a similar pattern of future behavior, we have that \( \frac{\Delta U}{\Delta S} \leq \frac{\Delta U}{\Delta S_{l+1}} \) because \( (1 - \beta)^t \) is decreasing in \( l \). This completes the proof. \( \square \)

Proposition 2.3. Within this deterministic framework, the individual \( i \) chooses to engage in safe sex if:

\[
P_0 = \frac{R_0}{S_0} = 1 = \frac{1 + \alpha}{1 - \rho} - \frac{(1 + \alpha)p\beta\rho^{l+1}}{(1-p)[1-\rho(1-\beta)]} - 1 \leq 0.
\]

Proof If protection is always used, the expected lifetime utility of \( i \) is given by:

\[
S_0 = \sum_{t=0}^{\infty} \rho^t (\theta + \alpha) = \theta + \alpha \frac{1}{1 - \rho}.
\]

Conversely, if the individual \( i \) engages in risky sex, his expected lifetime utility is given by:

\[
R_0 = \sum_{t=0}^{l+1} \rho^t (1 + \alpha) + (1 - p_j) \sum_{t=1}^{\infty} \rho^t (1 + \alpha) + p_j \sum_{t=1}^{\infty} \rho^t (1 + \alpha)(1 - \beta)^{t-l+1} = \frac{1 + \alpha}{1 - \rho} - \frac{(1 + \alpha)p\beta\rho^{l+1}}{(1-p)[1-\rho(1-\beta)]}.
\]

\( \square \)

Proposition 2.5. After the unplanned risky encounter in \( t = -1 \), the individual chooses safe sex if:

\[
P_1 = \frac{1 + \alpha}{1 - \rho} - \frac{(1 + \alpha)p\beta\rho^{l+1}}{(1-p)[1-\rho(1-\beta)]} - 1 \leq -\epsilon_0.
\]
Proof The proof is similar to that of proposition 2.3, but with a higher probability of being dead after \(t = x - 1\) as an unprotected encounter occurred at time \(t = -1\). If protection is always used for \(t \geq 0\), the expected lifetime utility of \(i\) is given by:

\[
S_1 = \sum_{i=0}^{t-2} \rho^i(\theta + \alpha)(1 - p_j) \sum_{i=0}^{\infty} \rho^i(\theta + \alpha) + p_j \sum_{i=0}^{\infty} \rho^i(\theta + \alpha)(1 - \beta) = \frac{\theta + \alpha}{1 - \rho} - \frac{(\theta + \alpha)p\beta\rho^{x-1}}{(1 - \rho)}.
\]

Conversely, if the individual engages in risky sex, his expected lifetime utility is given by:

\[
R_1 = \sum_{i=0}^{t-2} \rho^i(1 + \alpha)(1 - p_j) \sum_{i=0}^{\infty} \rho^i(1 + \alpha) + p_j \sum_{i=0}^{\infty} \rho^i(1 + \alpha)(1 - \beta)^{x-2} = \frac{1 + \alpha}{1 - \rho} - \frac{(1 + \alpha)p\beta\rho^{x-1}}{(1 - \rho)(1 - \rho(1 - \beta))}.
\]

Proposition 2.6. The propensity \(P_1\) to engage in risky sexual behavior after an unprotected sexual encounter is a U-shaped function of the expected transmission rate \(\beta\). The transmission rate which minimizes the propensity \(P_1\) is given by:

\[
\beta_1 = \frac{-(1 - \rho) + \sqrt{(1 - \rho)(1 - p\beta\rho^{x-1})}}{\rho(1 - p\beta^{x-2})}.
\]

Proof The derivative of \(P_1\) with respect to \(\beta\) is given by:

\[
\frac{\partial P_1}{\partial \beta} = \frac{p_j(1 + \alpha)\rho^{x-1}[\rho^2(1 - p\beta^{x-2})\beta^2 + 2\rho(1 - \rho)\beta - \rho(1 - \rho)]}{(\alpha + \theta)(1 - (1 - \beta)p)^2(1 - p\beta\rho^{x-1})^2}.
\]

All the terms of the derivative are positive except the term enclosed in square brackets at the numerator. This term is quadratic in \(\beta\) and admits one positive and one negative root. The derivative \(\frac{\partial P_1}{\partial \beta}\) has therefore the same roots, which are given by:

\[
-\rho(1 - \rho) \pm \sqrt{\rho^2(1 - \rho)(1 - p\beta^{x-1})}
\]

Between these two roots, the derivative is negative. The positive root is comprised between 0 and 1. Therefore, \(P_1\) is a U-shaped function of \(\beta\) for \(\beta \in [0, 1]\).

Proposition 3.1. If the individual engages in casual sex throughout his life, he chooses to use condoms in \(t = 0\) if:

\[
P_0^c = \frac{R_0^c}{S_0^c} - 1 = \frac{1 + \alpha}{1 - \rho} - \frac{(1 + \alpha)p\beta^{x-1}}{\rho(1 - (1 - \beta)p)} - 1 \leq -\epsilon_0.
\]

Proof If protection is always used, the expected lifetime utility of \(i\) is given by:

\[
S_0^c = \sum_{i=0}^{\infty} \rho^i(\theta + \alpha) = \frac{\theta + \alpha}{1 - \rho}.
\]

Conversely, if the individual engages in risky sex, his expected lifetime utility is given by:

\[
S_0^c = \sum_{i=0}^{\infty} \rho^i(\theta + \alpha) = \frac{\theta + \alpha}{1 - \rho}.
\]

Conversely, if the individual engages in risky sex, his expected lifetime utility is given by:
Proposition 3.3. After an unplanned risky encounter in \( t = -1 \), the individual \( i \) chooses safe sex if:

\[
P_{c1}^i = \frac{1 + \alpha}{1 - \rho} \left( \frac{(1 + \alpha) \beta p^{\lambda}}{1 - (1 - \rho)(1 - \beta p)} \right) \leq -\epsilon_0.
\]

Proof The proof is similar to that of proposition 3.1, but with a higher probability of being dead after \( t = \lambda - 1 \) as an unprotected encounter occurred at time \( t = -1 \). If protection is always used for \( t \geq 0 \), the expected lifetime utility of \( i \) is given by:

\[
S_{c1}^i = \sum_{t=0}^{\lambda-2} \rho^t (\theta + \alpha) + \sum_{t=\lambda-1}^{\infty} \rho^t (\theta + \alpha)(1 - \beta p)
\]

Conversely, if the individual \( i \) engages in risky sex, his expected lifetime utility is given by:

\[
R_{c1}^i = \sum_{t=0}^{\lambda-2} \rho^t (1 + \alpha) + \sum_{t=\lambda-1}^{\infty} \rho^t (1 + \alpha)(1 - \beta p)^{t+1} = \frac{1 + \alpha}{1 - \rho} \left( \frac{(1 + \alpha) \beta p^{\lambda}}{1 - (1 - \rho)(1 - \beta p)} \right).
\]

Proposition 3.4. The propensity \( P_{c1}^i \) to engage in risky sexual behavior after an unprotected sexual encounter is a U-shaped function of the expected transmission rate \( \beta \) if:

\[
p > \frac{-(1 - \rho) + \sqrt{(1 - \rho)(1 - \beta p^{\lambda+1})}}{\rho(1 - \beta p^{\lambda+2})}.
\]

In this case, the transmission rate which minimizes the propensity \( P_{c1}^i \) is given by:

\[
\beta_{c1}^* = \frac{-(1 - \rho) + \sqrt{(1 - \rho)(1 - pp^{\lambda+1})}}{pp(1 - pp^{\lambda+2})} = \frac{\beta_{c1}^p}{p}.
\]

If the condition 11 is not satisfied, the propensity \( P_{c1}^i \) is a strictly decreasing function of \( \beta \) with a minimum in \( \beta = 1 \).

Proof The derivative of \( P_{c1}^i \) with respect to \( \beta \) is given by:

\[
\frac{\partial P_{c1}^i}{\partial \beta} = \frac{p(1 + \alpha)\beta^{\lambda+1}|p^2(1 - \beta p^{\lambda+2})| + 2pp(1 - \rho)\beta - \rho(1 - \rho)}{(\alpha + \theta)(1 - (1 - \beta p)p^2(1 - \beta pp^{\lambda+1})^2)}.
\]
All the terms of the derivative are positive except the term enclosed in square brackets at the numerator. This term is quadratic in $\beta$ and admits one positive and one negative root. The derivative $\partial P_i / \partial \beta$ has therefore the same roots, which are given by:

$$ P_i = -\frac{(1 - \rho) + \sqrt{(1 - \rho)(1 - pp^{N+1})}}{pp(1 - pp^{N+2})} $$

Between these two roots, the derivative is negative. The positive root is comprised between 0 and 1 if:

$$ \rho > \frac{-\rho + \sqrt{1 - \rho(1 - pp^{N+1})}}{p(1 - pp^{N+2})} $$

If this condition is satisfied, $P_i^*$ is a U-shaped function of $\beta$. Otherwise, $P_i^*$ is a decreasing function of $\beta$. □

Appendix B. Supplementary propositions

**Proposition B.1.** After $N$ unplanned risky encounters with $j$ ($\lambda > N > 0$), the individual chooses safe sex at time $t = 0$ if:

$$ P_N = \frac{1 + \alpha}{1 + \rho} - \frac{(1 + \alpha) p \cdot \beta^0 \cdot (1 - \rho)}{1 - \rho(p(1 - 1 - \rho))} - 1 \leq -\epsilon_0. $$

**Proof** The proof is similar to the proof of proposition 2.5. If protection is always used, the expected lifetime utility of $i$ is given by:

$$ S_N = \sum_{t=0}^{N-1} \rho^t(\theta + \alpha) + (1 - p_j) \sum_{t=0}^{\infty} \rho^t(\theta + \alpha) + p_j \sum_{t=0}^{N-1} \rho^t(\theta + \alpha)(1 - \beta)^t + \sum_{j=1}^{\infty} \rho^t(\theta + \alpha)(1 - \beta)N $$

Conversely, if the individual $i$ engages in risky sex, his expected lifetime utility is given by:

$$ R_N = \sum_{t=0}^{N-1} \rho^t(1 + \alpha) + (1 - p_j) \sum_{t=0}^{\infty} \rho^t(1 + \alpha) + p_j \sum_{t=0}^{N-1} \rho^t(1 + \alpha)(1 - \beta)^t + \sum_{j=1}^{\infty} \rho^t(1 + \alpha)(1 - \beta)^t $$

$$ = \frac{1 + \alpha}{1 - \rho} - \frac{(1 + \alpha) p \cdot \beta^0 \cdot (1 - \rho)}{1 - \rho(p(1 - 1 - \rho))} \square $$

**Proposition B.2.** The propensity $P_N$ to engage in risky sexual behavior after $N > 0$ unprotected sexual encounters is a U-shaped function of the expected transmission rate $\beta$ ($\lambda > N > 0$). The transmission rate which minimizes the propensity $P_N$ is given by:

$$ \beta_N^* = \frac{-(N + 1)(1 - \rho) + \sqrt{(N + 1)^2(1 - \rho)^2 + 4p(1 - \rho)N(1 - pp^{N+1})}}{2Np(1 - pp^{N+1})} $$

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The proof is similar to the proof of proposition 2.6. The derivative of \( P_N \) with respect to \( \beta \) is given by:

\[
\frac{\partial P_N}{\partial \beta} = \frac{p_j(1 + \alpha)p^{\beta + N}(1 - \beta)^N - (N + 1)p^\beta(1 - \beta)(1 - \beta)}{(1 + \theta)(1 - \beta)(1 - \beta)p + \beta p^\beta(1 - \beta)^N(1 - \beta)}.
\]

All the terms of the derivative are positive except the term enclosed in square brackets at the numerator. This term is quadratic in \( \beta \) and admits one positive and one negative root. The derivative \( \frac{\partial P_N}{\partial \beta} \) has therefore the same roots, which are given by:

\[
-(N + 1)(1 - \rho) \pm \sqrt{(N + 1)^2(1 - \rho)^2 + 4p(1 - \rho)N(1 - \rho^N(1 - \rho))}
\]

\[
2Np(1 - pp^N(1 - \rho))
\]

Between these two roots, the derivative is negative. The positive root is comprised between 0 and 1. Therefore, \( P_N \) is a U-shaped function of \( \beta \) for \( \beta \in [0, 1] \)

Proposition B.3. After \( N \) unplanned risky encounters with \( j (\lambda > N > 0) \), the individual chooses safe sex at time \( t = 0 \) if:

\[
P^*_N = \frac{\frac{1}{1 + \alpha} - \frac{1 + \alpha p^N}{1 + \beta}}{\frac{1}{1 + \alpha} - \frac{1 + \alpha p}{1 - \beta}} - 1 \leq -\epsilon_0.
\]

Proof The proof is similar to the proof of proposition 3.3. If protection is always used, the expected lifetime utility of \( i \) is given by:

\[
S_N^i = \sum_{t=0}^{N-1} \rho^t(\theta + \alpha) + \sum_{t=N}^{\infty} \rho^t(\theta + \alpha)(1 - \beta p)^{t-N} + \sum_{t=0}^{\infty} \rho^t(\theta + \alpha)(1 - \beta p)^N
\]

\[
\leq \frac{\theta + \alpha}{1 - \rho} = \frac{(\theta + \alpha)p^{N} - (1 - \beta p)^N}{(1 - \rho)(1 - (1 - \beta p))}.
\]

Conversely, if the individual \( i \) engages in risky sex, his expected lifetime utility is given by:

\[
R^*_N = \sum_{t=0}^{N-1} \rho^t(1 + \alpha) + \sum_{t=N}^{\infty} \rho^t(1 + \alpha)(1 - \beta p)^{t-N} + \sum_{t=0}^{\infty} \rho^t(1 + \alpha)(1 - \beta p)^N
\]

\[
\leq \frac{1 + \alpha}{1 - \rho} = \frac{(1 + \alpha)p^{N} - (1 - \beta p)^N}{(1 - \rho)(1 - (1 - \beta p))}.
\]

Proposition B.4. When the individual \( i \) engages in casual sex, the propensity \( P^*_N \) to engage in risky sexual behavior after \( N > 0 \) unprotected sexual encounter is a U-shaped function of the expected transmission rate \( \beta \) if:

\[
p > \frac{-(N + 1)(1 - \rho) + \sqrt{(N + 1)^2(1 - \rho)^2 + 4p(1 - \rho)N(1 - \rho^N(1 - \rho))}}{2Np(1 - \rho^N(1 - \rho))}
\]

(B.1)

If this condition is satisfied, the transmission rate which minimizes the propensity \( P^*_N \) is given by:

\[
\beta^*_N = \frac{-(N + 1)(1 - \rho) + \sqrt{(N + 1)^2(1 - \rho)^2 + 4p(1 - \rho)N(1 - \rho^N(1 - \rho))}}{2Np(1 - \rho^N(1 - \rho))} = \frac{\beta^*_N}{p}
\]

If condition B.1 is not satisfied, \( P^*_N \) is a decreasing function of \( \beta \).
Proof The proof is similar to the proof of proposition 3.4. The derivative of $P_N^c$ with respect to $\beta$ is given by:

$$\frac{\partial P_N^c}{\partial \beta} = \frac{p(1 + \alpha)\rho^{i+N}(1 - \beta p)^{N-1}[N\rho^{N+1}\rho^2(1 - \rho^{1-N-1})\rho^2 + p(N + 1)\rho^N(1 - \rho)\beta - \rho^N(1 - \rho)]}{(\alpha + \theta)[\rho^N(1 - (1 - \beta p)\rho) - \beta pp^i(1 - \rho^N(1 - \beta p)\rho)]^2}.$$ 

All the terms of the derivative are positive except the term enclosed in square brackets at the numerator. This term is quadratic in $\beta$ and admits one positive and one negative root. The derivative $\frac{\partial P_N^c}{\partial \beta}$ has therefore the same roots, which are given by:

$$\frac{-(N + 1)(1 - \rho) \pm \sqrt{(N + 1)^2(1 - \rho)^2 + 4\rho(1 - \rho)N(1 - pp^{t-N-1})}}{2Np(1 - pp^{t-N-1})}.$$ 

Between these two roots, the derivative is negative. The positive root is comprised between 0 and 1 if:

$$p > \frac{-(N + 1)(1 - \rho) + \sqrt{(N + 1)^2(1 - \rho)^2 + 4\rho(1 - \rho)N(1 - \rho^{1-N-1})}}{2Np(1 - \rho^{t-N-1})}.$$ 

If this condition is satisfied, $P_N^c$ is a U-shaped function of $\beta$. Otherwise, $P_N^c$ is a decreasing function of $\beta$. □